Mathematical Reasoning

115 minutes, ~45 questions – first 5 questions without calculator

The Mathematical Reasoning Test measures your understanding of the applications of arithmetic and algebraic reasoning. The questions are in the forms of multiple-choice, fill-in-the-blank, drop-down, drag-and-drop, and graphing. The questions may contain a combination of text, graphics, charts, diagrams, number lines, and coordinate grids. Most questions will be word problems. You need a baseline understanding of arithmetic and algebraic reasoning to pass the Mathematical Reasoning Test.

Areas covered on the test:

- 45% Arithmetic
- 55% Algebraic Reasoning

Tools you may use:

- TI-30 XS Calculator and Calculator Reference Sheet
- Formula Sheet
- Wipe-off board and dry erase marker

Ratio

A ratio is a way of comparing two values. They are written like they are read, the number that is read first will occur first (or in the numerator) in the ratio.

Ex: 3 to 1 , 3:1 , and $\frac{3}{1}$ are all ways we can write a ratio that means for every 3 of one item there is 1 of another. Similar to fractions, ratios can be simplified into lowest terms.

I work in a bookstore, and in one day I ring up 50 purchases. 15 of those purchases are cash, the rest are done with a credit or debit card. The following are potential ratio comparisons for this question:

Credit Card Purchases to *Total Purchases* → 35 to 50, 35:50 or $\frac{35}{50}$

Simplified to lowest terms: 7 to 10, 7:10 or $\frac{7}{10}$

Cash Purchases to *Card Purchases* → 15 to 35, 15:35 , or $\frac{15}{35}$

Simplified to lowest terms: 3 to 7, 3:7 , or $\frac{3}{7}$

Cash Purchases to *Total Purchases* → 15 to 50, 15:50 , or $\frac{15}{50}$

Simplified to lowest terms: 3 to 10, 3:10 or $\frac{3}{10}$

Distance and Cost

Distance and cost formulas have 3 parts to them. To solve a distance or cost formula, you identify what two things you have in order to solve for the third.

Distance = Rate you travel (speed) x Time it takes (hours) D = R x T

So, if you know the distance and the rate, you can solve for time. If you know the rate and the time you can solve for the distance. If you know the distance and the time you can solve for the rate.

D = R x T D / R = T D / T = R

Total Cost = Price per item (rate) x Number of items being purchased C = R x N

So, if you know the price per item and the number of items being purchased you can solve for the total cost. If you know the total cost and the price per item, you can solve for the number of items. If you know the total cost and the number of items, you can solve for the price per item.

C = N x R C / R = N C / N = R

Exponents

Exponents or "powers" are a way of showing repeated multiplication. It has two parts, the base, or the big number, and the exponent, or the superscript.

We read 4^6 as "four to the power of six" or "four to the sixth power". In this example, 4 is the base and 6 is the exponent. The exponent indicates how many times the base is multiplied to itself, so 4^6 = 4 x 4 x 4 x 4 x 4 x 4 or 4 multiplied to itself 6 times, the product of which is 4,096.

RULES:

<u>Any</u> base to the power of 0 equals 1 □ 9^0 =1

<u>Any</u> base to the power of 1 equals itself □ 6^1 =6

<u>Any</u> number raised to a negative power equals a fraction with a numerator of 1 □

$$5^{-2} \;\square\; \frac{1}{5^2} \;\square\; \frac{1}{5\,x\,5} \;\square\; \frac{1}{25}$$

Roots

Roots are a way of finding "What number multiplied to itself (x number of times) is equal to the number found under the radical sign ($\sqrt{X}$). The number below the radical sign is called the radicand. The number out front of the radical sign is called the index. The index tells us how many times a number was multiplied to itself to give us the radicand. *If there is no index, the index is automatically 2.

$\sqrt{25}$ In this example, 25 is the radicand and 2 is the index (when there is no index number it is automatically 2), so to solve this problem you are looking for what two numbers that are the same that multiply together to make 25 □

______ x ______ = 25 since 5^2 = 25, then $\sqrt{25}$ = 5

$\sqrt[3]{8}$ In this example, 8 is the radicand and 3 is the index, so to solve this problem you are looking for what three numbers that are the same that multiply together to make 8 □

______ x ______ x ______ = 8 since 2^3 = 8 , then $\sqrt[3]{8}$ = 2 *This is an inverse relationship!

Scientific Notation

This is a way of using powers of 10 to write numbers that are very large or very small. A scientific notation is a way of expressing a product of a number between 1 and 10 multiplied to 10 to a certain power.

(1-10) x $10^{\#}$

Positive exponents indicate this number is large, and the decimal has to move from its position in scientific notation form to the right to expand the number of decimal places that the exponent indicates:

1.564 x 10^{5} → 156,400.00 *The decimal place moved 5 places to the right*

Negative exponents indicate this number is small, and the decimal has to move from its position in scientific notion form to the left to expand the number of decimal places that the exponent indicates:

3.61 x 10^{-6} → 0.00000361 *The decimal place moved 6 places to the left*

To put a number into scientific notation, first recognize if you have a large number or a small (decimal) number. The decimal will be moved create a number between 1-10.

3,457,000,000.00 → 3.457 10^{9} 0.000004431 → 4.431 x 10^{-6}

Order of Operations

When you are working on a math problem that contains more than one operation, the answer will vary depending on how these operations are performed. Therefore, we follow the Order of Operations when solving problems, performing operations in the following order:

1) P – Parentheses (or other grouping symbols like brackets, absolute value bars, division fraction bar)

For parentheses inside of parentheses (or brackets, bars etc.) begin PEMDAS over again

2) E – Exponents and roots

3) M / D – Multiplication and division, working left to right

4) A / S – Addition and subtraction, working left to right

PARENTHESES → $(5+7)^{2}$ – (8)(3)

EXPONENTS → $(12)^{2}$ – (8)(3)

MULTIPLICATION → (144) – (8)(3)

SUBTRACTION → 144 – 24

ANSWER → = 120

Percent

Percent means “per every 100,” so if you had 3% you have 3 parts out of every 100 parts

Base = The whole quantity **Part** = Portion of the base **Rate** = % the part describes

$\frac{12}{48}$ **=** ***0.25*** → ***25%***

In this proportion, 12 is the part of the whole, 48 is the base (the whole), and 25% is the rate, or the percent that 12 represents of the whole (100%). As long as you have two, you can solve for the missing third value.

BASE X RATE = PART PART / BASE = RATE PART / RATE = BASE

A basketball team plays in a tournament where they win 9 games and lose 3 games. What percent of their games did they win? Round your answer to the nearest whole percent

What is the PART? 9

What is the BASE? 12 (total games)

= 0.75 75%

You have the Part and Base, to find the rate apply the formula:

PART / BASE = RATE 9 / 12 = 0.75 → 75% is the rate

Central Tendency

Central Tendency describes ways of analyzing raw data scores to determine the middle values.

Mean (average) – the arithmetic average of the values

Median – the middle value of a data set

Mode – the value that most often occurs in a data set

Range - the area of variation between the upper and lower values

Example Data Set **: 76, 82, 75, 87, 80, 82, 79** Find Central Tendency, round to the nearest tenth.

To find the mean in this data set, find the sum of the numbers and divide the sum by how many numbers were added together

$\frac{76 + 82 + 75 + 87 + 80 + 82 + 79}{7}$ = 80.1428 → 80.1

To find the median in this data set, arrange the numbers from smallest to greatest, and cross out numbers on the left and right side until you have the single middle value. If you have two middle values in a data set with an even number of values, take the average of the two.

~~75~~, ~~76~~, ~~79~~, 80, ~~82~~, ~~82~~, ~~87~~ → 80 is the median of this data set

To find the mode, we look for the number that most often occurs in our data set.

76, 82, 75, 87, 80, 82, 79 ⍰ 82 is the value most often occurring in this data set

To find the range, take the difference between the highest and lowest values in the set.

87 – 76 = 11 ⍰ 11 is the range of this data set

Probability

Probability is the likelihood of an event to occur. It compares favorable outcomes (what we are looking for) to the total outcomes (everything that has happened).

Theoretical probability tests the likelihood an event may occur mathematically, it predicts what will happen.

Experimental probability describes the likelihood of an event based on what happened in an actual experiment.

The *theoretical probability of rolling a 1 on a 6-sided die is* $\frac{1}{6}$, but if I perform an experiment where I roll the die 10 times and 3 of those times the die lands on 1, I have an *experimental probability of* $\frac{3}{10}$ to describe the favorable outcomes that have occurred compared to the total outcomes.

Probability is indicated by a value between 0 and 1, with 0 meaning the event *will never* occur and 1 meaning the event *will always* occur.

These values can be written as fractions, decimals or percentages.

> Ex: A probability of $\frac{1}{2}$ can be written as 0.5 or 50% (see fraction-decimal equivalency, pg. 250)
>
> This indicates that 1 out of every 2 times an event happens you are expected to have a favorable outcome. The probability of flipping heads (or a tails) on a quarter after one toss is represented by the example- You have two potential outcomes, heads is one of the two outcomes.

You roll a single die 3 times. These events are **independent** (one outcome does not affect the other).

You draw cards from a deck and do not replace them back into the deck. These events are **dependent** (the outcome of the succeeding events depends on the outcome of the first).

Simple probability expressed the likelihood of one event occurring. Compound probability describes the chances of more than one separate event happening. Compound probability is solved by multiplying the probabilities of the individual events together.

Algebraic Expressions, Equations, and Inequalities

An **algebraic expression** is built from numbers, variables, and operations. A **polynomia**l is a type of expression made up of a sum of two or more terms. A term can be a constant or a variable with a coefficient.

constant: a specific number.
variable: a letter used to represent an unknown value.
coefficient: a number multiplied times a variable.
operations: addition, subtraction, multiplication, division, raising to a power.

ex. $3x^2 + 2x + 5$ 3 terms: variable is x, coefficients are 3 and 2, constant is 5

- Translate words into an algebraic expression: 5 less than the quotient of 10 and a number $\rightarrow$ $\frac{10}{x} - 5$

- Simplify an expression:

$3x - 6(x - 9)$

distribute: $3x - 6x + 54$

combine like terms: $-3x + 54$

- Evaluate an expression:

Find the value of $(x + y)^2 - 10$ when $x = 2$ and $y = 4$

substitute: $(2 + 4)^2 - 10$

solve: $6^2 - 10$

$36 - 10$

26

- Add, subtract, multiply, and divide polynomials:

Simplify:

$\frac{8b(9ab+8a)-4ab}{12}$

$\frac{72ab^2+64ab-4ab}{12}$

$\frac{72ab^2+60ab}{12}$

$6ab^2 + 5ab$

An **equation** is a statement that two expressions are equal. To solve an equation, use inverse operations to isolate the variable.

- Solve an equation:

$3x - 20 = 130$

add 20 to both sides of the equation: $3x - 20 + 20 = 130 + 20$

$3x = 150$

divide both sides of the equation by 3: $\frac{3x}{3} = \frac{150}{3}$

$x = 50$

An **inequality** is a statement that connects two unequal expressions with one of the following relationships.

less than $<$

less than or equal to $\leq$
greater than $>$
greater than or equal to $\geq$

An inequality is solved like an equation with one exception – when multiplying or dividing by a negative number, you must reverse the inequality sign.

Solve the inequality and graph on a number line:

$$-4(x+2) < 24$$

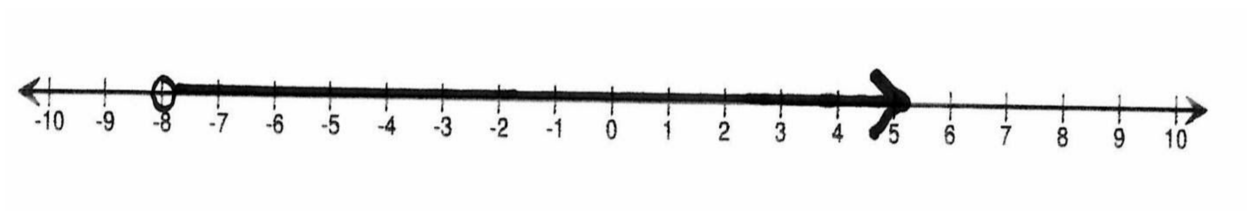
distribute: $-4x - 8 < 24$

add 8 to both sides: $-4x - 8 + 8 < 24 + 8$

$$-4x < 32$$

divide both sides by -4 and reverse sign: $\frac{-4x}{-4} > \frac{32}{-4}$

$$x > -8$$



<u>Linear Equations and Their Graphs</u>

A linear equation is one that can be written in the form y = mx + b and whose graph is a straight line.

y = mx + b is called **the slope-intercept form** of the equation. In this form:

m represents the slope of the line when graphed.
b represents the point where to line crosses the y-axis called the y-intercept. (0 , b)
x and **y** represent the solutions to the equation and the coordinates of points on the line. (x , y).

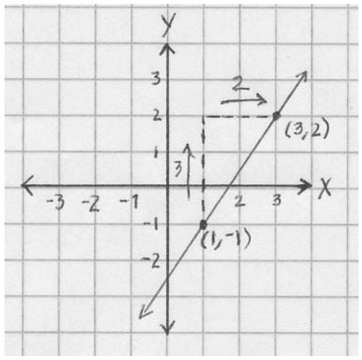
Slope is a measure of the steepness of the line. It gives the rate at which values of the equation are increasing or decreasing.

positive slope – the lines rises to the right of the graph.
negative slope – the line falls to the right of the graph.

Parallel lines have the same slope.
Perpendicular lines have slopes that are negative reciprocals.

There are two common ways to find slope:

- On the graph, count the units between any two points on the line and make a fraction.



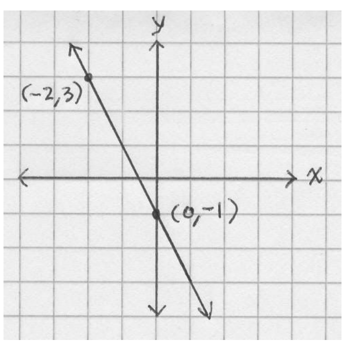
$$m = \frac{\textit{vertical change}}{\textit{horizontal change}} = \frac{3}{2}$$

- Use the coordinates of two points (x_1, y_1) and (x_2, y_2) in the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

For the line through points (1, -1) and (3, 2): $m = \frac{2-(-1)}{3-1} = \frac{2+1}{3-1} = \frac{3}{2}$

Use the graph of a line to write its equation.



The slope between the two points is down 4 units and over 2, so

$$m = \frac{-4}{2} = -2$$

The line crosses the y-axis at the point (0, -1), so *b* = -1
Substitute those values to get the equation of the line:

$$y = mx + b \quad \rightarrow \quad y = -2x - 1$$

Solve a linear equation for y to get it in slope-intercept form:

$$3x + 4y = 8$$
$$3x - 3x + 4y = 8 - 3x$$
$$4y = -3x + 8$$
$$\frac{4y}{4} = \frac{-3x}{4} + \frac{8}{4}$$
$$y = -\frac{3}{4}x + 2$$

Functions

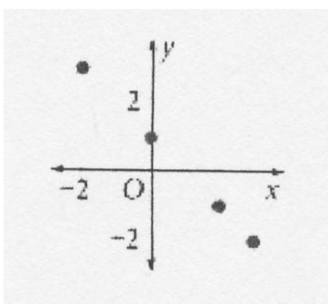
A function is a special type of relation that guarantees each input value (x) is paired with exactly one output value (y). It can be represented with an equation, a graph, or a list of ordered pairs.

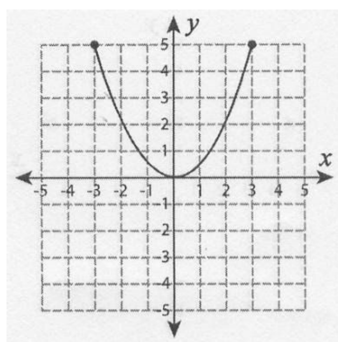
- To indicate that an equation is a function, we use a special notation, replacing y with **f(x)**.
  (We say this "f of x")

$$y = 3x^2 - 7 \rightarrow f(x) = 3x^2 - 7$$

- To identify a function from a graph, use the Vertical Line Test - no vertical line can pass through more than one point on the graph of a function.
- To identify a function from an x-y list make sure no x-values are repeated with different y-values.

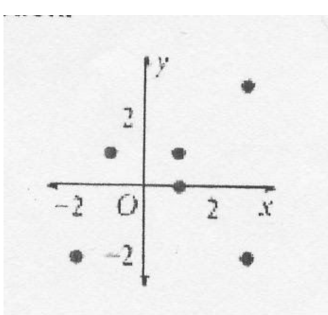
These represent functions:

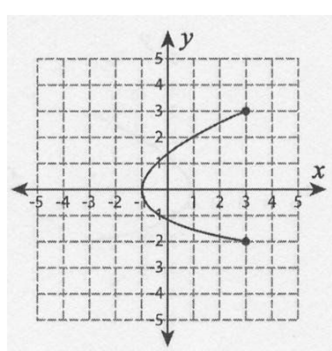




x	y
−5	−4
−4	−4
−3	−2
0	−1

These do not represent functions:





x	y
−5	4
5	−1
5	3
1	4